MAS1107: Discrete Mathematics and Probability: Assignment 1

Solutions must be submitted by Wednesday 5 March

1. Write down the elements of the following sets:

$$A = \{n \in \mathbf{Z} : -2 \leq n \leq 4\}; \qquad B = \{n(n+1)(n+2)/6 : n \in A\};$$
$$C = \{n \in A : n \text{ is even}\}.$$

[9 marks]

2. 100 students were surveyed about their sporting activities. Each of them played at least one of the following three games: football, cricket and rugby. 56 play football, 51 play cricket, 55 play rugby, 24 play football and cricket, 21 play football and rugby, 28 play cricket and rugby. How many play all three games? [10 marks]

3. In a variant of poker each player is dealt a hand of six cards from a standard pack. How many hands are there of each of the following types?

(i) *three pairs*: two cards of the same value, another two cards of a second value, and two more of a third value. value, for instance, $Q\heartsuit$ $Q\clubsuit$ $10\spadesuit$ $10\clubsuit$ $9\diamondsuit$ $9\clubsuit$;

(ii) *two threes*: three cards of the same value, and another three of a second value, for instance, $7\heartsuit$ $7\clubsuit$ $7\spadesuit$ $3\heartsuit$ $3\diamondsuit$ $3\clubsuit$;

(iii) *super full house*: four cards of the same value, and two more of a second value, for instance, $3\diamondsuit$ $3\clubsuit$ $3\spadesuit$ $3\heartsuit$ $K\diamondsuit$ $K\clubsuit$. [18 marks]

4. How many anagrams has the word DISTRESSED? How many of them do not contain either of the words SIDE or DRESS? [16 marks]

5. Sum the arithmetic progression $\sum_{n=2}^{50}(10n+7)$ and the geometric progression $\sum_{n=0}^{8}(-3)^n$. [12 marks]

6. Prove by induction that

$$\sum_{k=1}^{n} k2^{k-1} = 1 + (n-1)2^n.$$

[15 marks]

7. Prove by induction that if $n \in \mathbf{N}$ then either $n = 3a$, $n = 3a+1$ or $n = 3a+2$ for some integer a. [20 marks]

RJC 6/2/2008