MAS1107: Discrete Mathematics and Probability: Assignment 2

Solutions must be submitted by Wednesday 30 April
Please give numerical answers correct to three significant figures

1. Let A, B and C be events. You are given that $P(A) = 0 \cdot 5$, $P(B) = 0 \cdot 46$, $P(C) = 0 \cdot 58$, $P(A \cap B) = 0 \cdot 2$, $P(A \cap C) = 0 \cdot 28$, $P(B \cap C) = 0 \cdot 3$ and $P(A \cap B \cap C) = 0 \cdot 11$. Find:

   (i) $P(A \cup B \cup C)$; (ii) $P(A \mid B)$; (iii) $P(B \mid A)$; (iv) $P(A \mid B \cup C)$; (v) $P(A \mid C^c)$. [10 marks]

2. You shuffle a standard pack of cards and deal three cards in a row.

   Let A be the event that the left card is a spade, B be the event that exactly two of the three cards are aces and C be the event that the right card is a king. Find:

   (i) $P(A)$; (ii) $P(B)$; (iii) $P(B \mid A)$; (iv) $P(C)$; (v) $P(B \mid C)$; (vi) $P(C \mid B)$. [24 marks]

3. There are three widget factories, in Abergavenny, Bedford and Cumbernauld. They produce respectively 35%, 28% and 37% of Widgets Unlimited's output. The failure rate of the widgets produced in the three factories are 3%, 6% and 4% respectively. A widget picked at random in the company's central warehouse is found to be defective. What is the probability that it was produced in (i) Abergavenny? (ii) Bedford? (iii) Cumbernauld? [15 marks]

4. You toss a fair coin repeatedly until you get three heads. What is the probability that you get the third head on (i) the fifth toss? (ii) the sixth toss? (iii) the seventh toss? [15 marks]

5. A discrete random variable has range space $\{1, 2, \ldots, n\}$ and satisfies $P(X = j) = j/c$ for some number c. Find c, and then find $E(X)$, $E(X^2)$, $E(1/X)$ and $\mathrm{Var}(X)$. [20 marks]

6. On a certain Saturday twenty million people enter the lottery. Assuming each chooses their numbers randomly and that the probability that a given ticket wins the jackpot is 1/13983816 find, (to three significant figures) the probability that (i) no one wins the jackpot, (ii) one person wins the jackpot, (iii) two persons win the jackpot, (iv) three or more persons win the jackpot. [16 marks]

   [Hint: use the Poisson distribution.]

RJC 10/3/2008