

Vectors and matrices: matrices in MAPLE

This is a short introduction, with exercises, to using matrices in MAPLE. I am not asking for these exercises to be submitted; so in a sense they are “optional” but I hope that you all do them sooner or later.

To manipulate matrices in MAPLE you need to load the `linalg` package.

```
> with(linalg):
```

This has to be done in every session you use matrices. The `matrix` command enters a matrix; for instance

```
> A:=matrix([[1,2,3],[-2,-4,0]]);
```

assigns the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & -4 & 0 \end{pmatrix}$$

to the variable A . To look at a matrix A don't type

```
> A;
```

but instead

```
> print(A);
```

MAPLE is somewhat awkward when doing matrix arithmetic. The obvious commands don't work; you have to surround them with the `evalm` command, and multiplication must be denoted by `&*` rather than `*` for some daft reason. As an example,

```
> B1:=matrix([[1,2],[3,4]]);
```

```
> B2:=matrix([[0,1],[-2,1]]);
```

```
> evalm(B1 + B2);
```

```
> evalm(B1 - B2);
```

```
> evalm(B1 &* B2);
```

```
> evalm(B2 &* B1);
```

evaluates the sum, difference and both products of a pair of typical 2-by-2 matrices. Similarly to do matrix powers, you need `&^` not `^`:

```
> evalm(B1 &^ 5);
```

Negative powers work for nonsingular matrices:

```
> evalm(B1 &^ (-1));
```

as well as

```
> inverse(B1);
```

finds the inverse of the matrix B_1 . Of course, one can do more complex calculations:

```
> evalm((B1 - B2)&^ 3 - B2 &^ (-1) &* (B1 + B2) &^ 5);
```

EXERCISE 1 Let

$$A = \begin{pmatrix} 0 & 1 \\ 3 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} 1 & -1 & 1 \\ 0 & -4 & 3 \end{pmatrix}.$$

Find $A + B$, AB , B^{-1} , $B^{-1}AB$, A^{10} , $(A - B)^6C$.

Elementary row (and column) operations can be done too.

```
> addrow(A,2,3,-4);
```

adds -4 times row 2 of A to row 3 of A ,

```
> mulrow(A,1,-1/2);
```

multiplies row 1 of A by $-1/2$ and

```
> swaprow(A,1,3);
```

swaps rows 1 and 3 of A .

EXERCISE 2 Use row operations to find the reduced row echelon form of

$$\begin{pmatrix} 2 & -4 & 6 & -1 \\ 1 & 8 & -2 & 3 \\ 1 & 2 & 1 & 1 \end{pmatrix}.$$

EXERCISE 3 Use row operations to find the inverse of

$$\begin{pmatrix} 0 & 2 & -1 & 3 \\ 3 & 1 & -1 & 0 \\ 2 & 4 & 5 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix}.$$

Determinants are computed by the `det` command:

```
> det(B1);
```

EXERCISE 4 Find the following determinants:

$$\begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{vmatrix}, \quad \begin{vmatrix} 1 & 1 & 1 & 1 \\ a & b & c & d \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \end{vmatrix}, \quad \begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix}.$$

To find the characteristic equation of a square matrix A use

```
> charpoly(A,t);
```

If MAPLE can solve the characteristic equation, it can compute the eigenvalues

and eigenvectors directly:

```
> eigenvalues(A);
```

```
> eigenvectors(A);
```

The output for `eigenvalues` is pretty self-explanatory, but that for `eigenvectors` is rather cryptic. As a simple example, let's start with

```
> A:=matrix([[1,0,0,0],[0,2,0,0],[0,0,2,1],[0,0,0,2]]);
```

Then

```
> eigenvalues(A);
```

returns

1, 2, 2, 2

showing, obviously enough, that A has eigenvalues 1 (unrepeated) and 2 (repeated three times). But

```
> eigenvectors(A);
```

returns

[2, 3, [0, 1, 0, 0], [0, 0, 1, 0]], [1, 1, [1, 0, 0, 0]].

This means that A has an eigenvalue 2 with algebraic multiplicity 3 and with general eigenvector

$$x \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + y \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix},$$

and that A has an eigenvalue 1 with algebraic multiplicity 1 and with general eigenvector

$$z \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

EXERCISE 5 Find the characteristic polynomial, eigenvalues and eigenvectors of

$$\begin{pmatrix} 3 & -1 & -3 & -4 & 2 \\ 1 & -5 & 0 & -1 & 8 \\ -2 & 2 & 2 & 2 & -4 \\ 2 & -2 & -3 & -3 & 4 \\ 1 & -4 & 0 & -1 & 7 \end{pmatrix}.$$