Vortex statistics in a simplified baroclinic model

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The model

Start from two-layer QG equations on a β-*channel*:

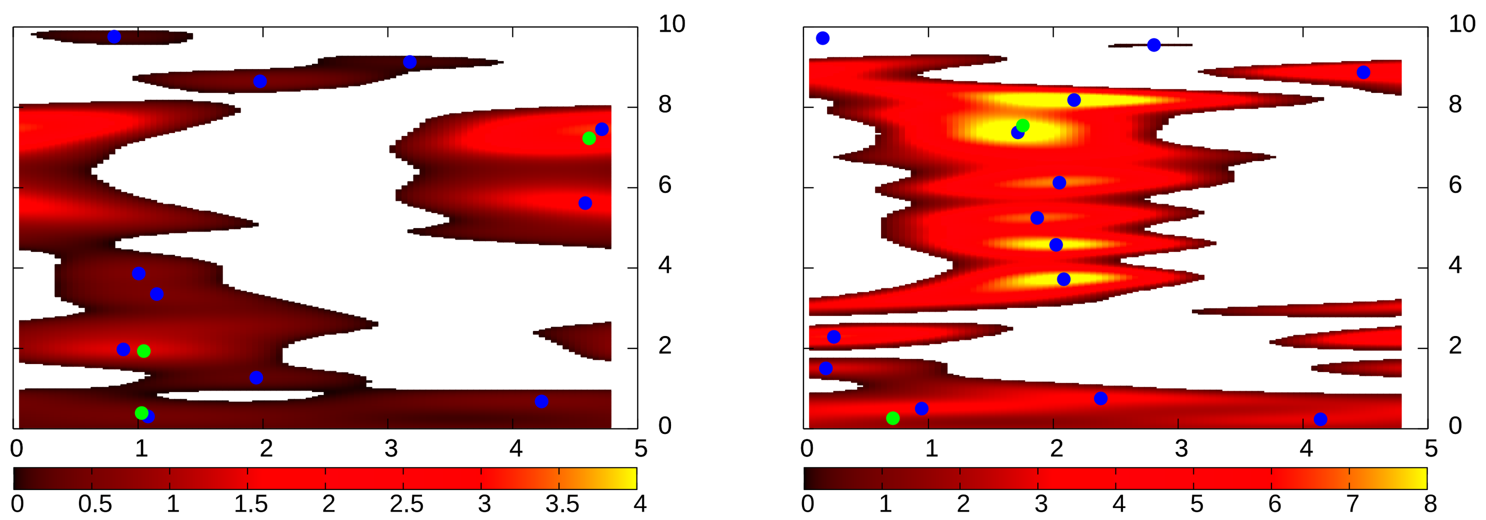
$$\partial_t \Delta_H \tau - \frac{2}{H_2^2}\partial_t \tau + J\left(\tau, \Delta_H \phi + \beta y + \frac{2}{H_2^2}\phi\right) + J(\phi, \Delta_H \tau) = \frac{2\nu_E}{H_2^2}\Delta_H(\phi - \tau) - \frac{2\kappa}{H_2^2}\Delta_H \tau + \frac{2\nu_N}{H_2^2}(\tau - \tau^\star), \quad (1)$$

$$\partial_t \Delta_H \phi + J(\phi, \Delta_H \phi + \beta y) + J(\tau, \Delta_H \tau) = -\frac{2\nu_E}{H_2^2}\Delta_H(\phi - \tau). \quad (2)$$

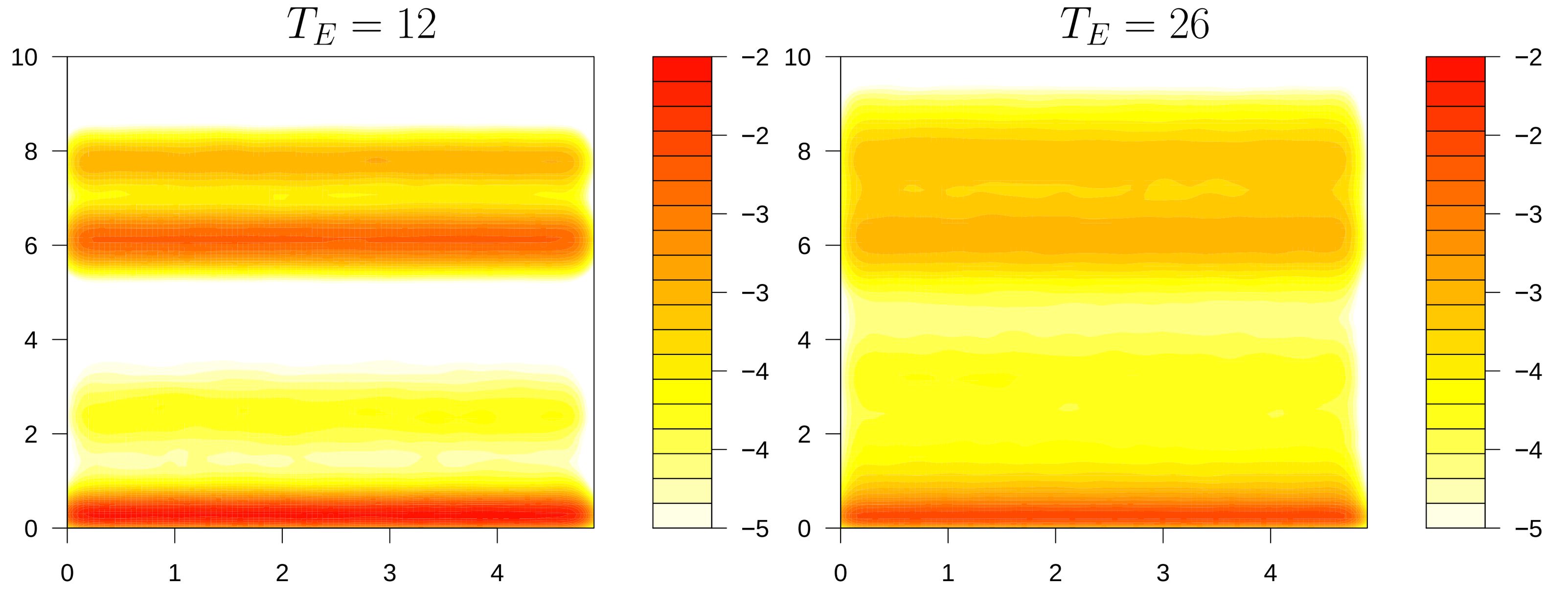
zonally periodic domain $(x, y) \in [0, L_x] \times [0, L_y]$; Δ_H: horizontal Laplacian;
$\tau = \frac{\psi_1 - \psi_3}{2}$, $\phi = \frac{\psi_1 + \psi_3}{2}$: baroclinic/barotropic compon. of streamfunction;
ν_E viscous-like Ekman pumping; κ heat diffusion;
ν_N: Newtonian relaxation towards temperature forcing profile τ^*.
Parameter T_E controls amplitude of baroclinic forcing τ^*.
Fourier-expand ϕ, τ: retain modes 0,3 in zonal direction x, modes 1-32 in meridional dir. $y \rightsquigarrow$ 1-wave/zonal wind model

The vortices

Vortex centers = minima of streamfunction ($\Longleftrightarrow$ cyclonic wind circulation)



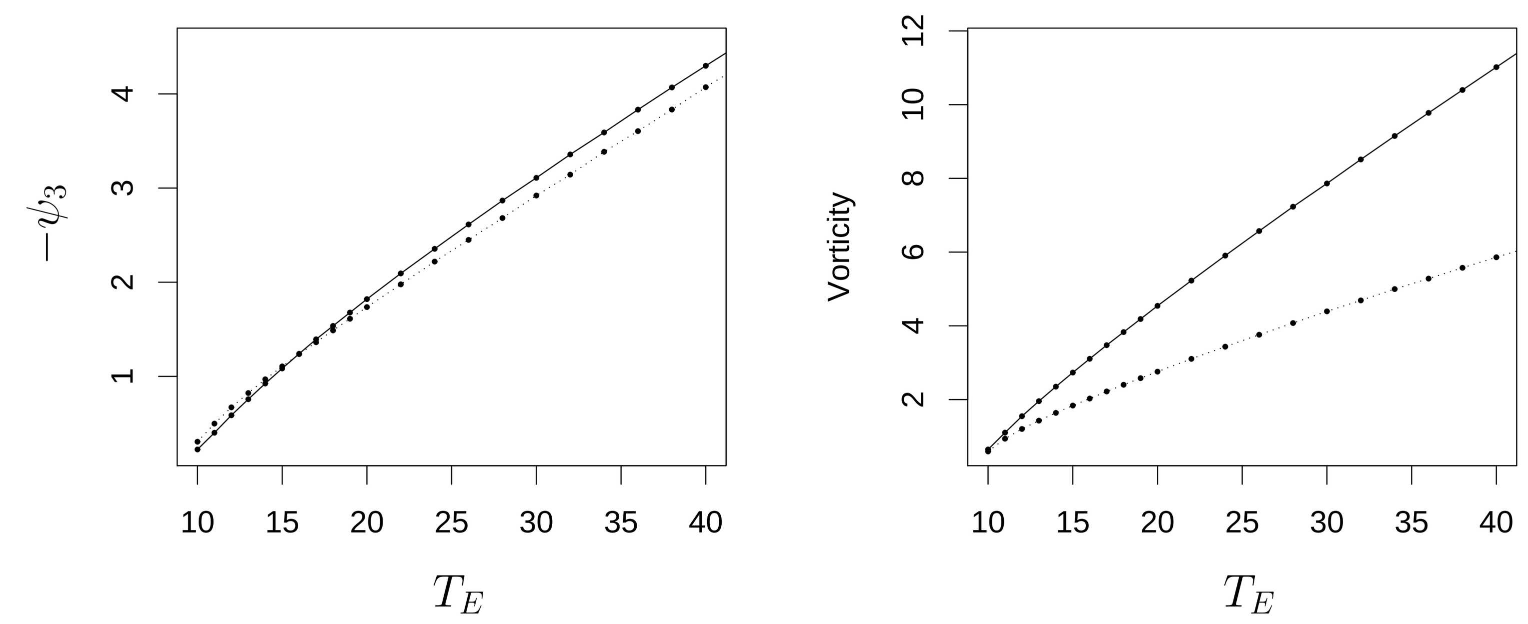
Positive relative vorticity in lower layer. Many vorticity maxima (blue dots) typically coexist with fewer minima of streamfunction ψ_3 (green).



Smoothed density of vortex centres in lower layer.
Small T_E: a *vortex jet* exists in northern half of channel.
Large T_E: distribution of vortices is latitudinally smoother

Smooth dependence on T_E

Several properties of baroclinic model depend fairly smoothly on T_E.
Previous studies: smooth scaling laws for Lyapunov exponents, dimension of chaotic attractor, extreme value of total energy.
Presently: we find smooth laws for quantities related to vortices.

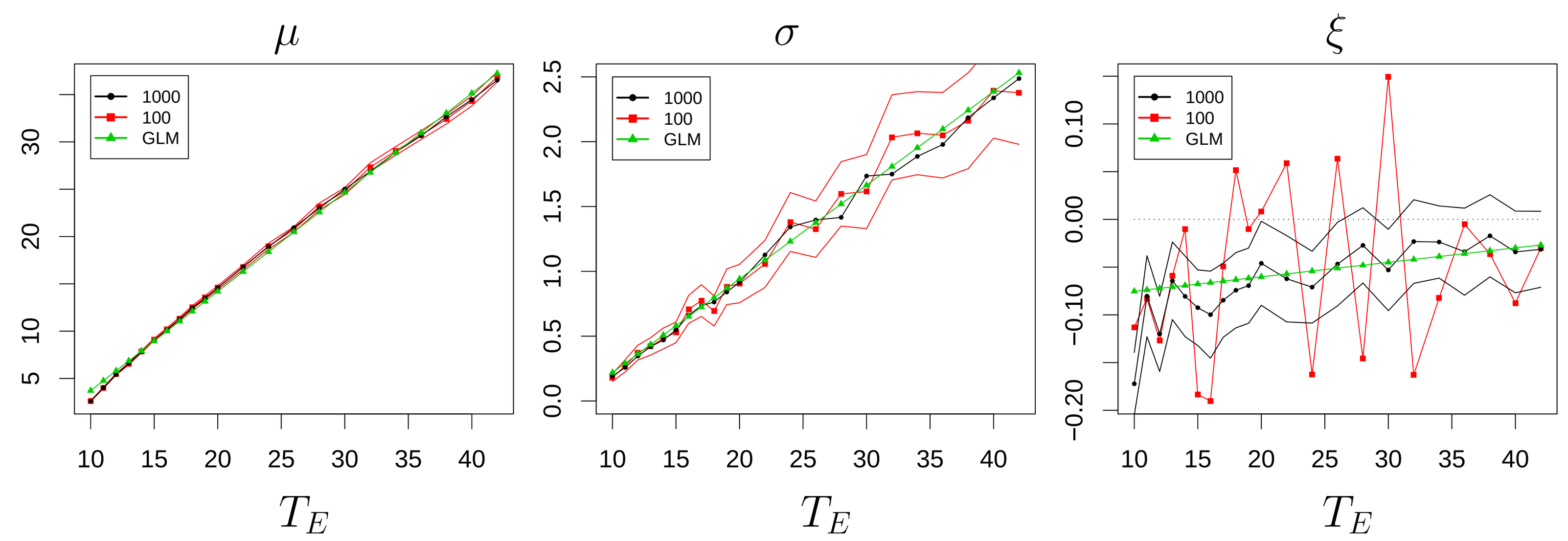


Left: mean/variance (solid/dashed lines) of lower layer streamfunction ψ_3 (mult. by -1) at vortex centres. Right: same for vorticity at vortex centres.

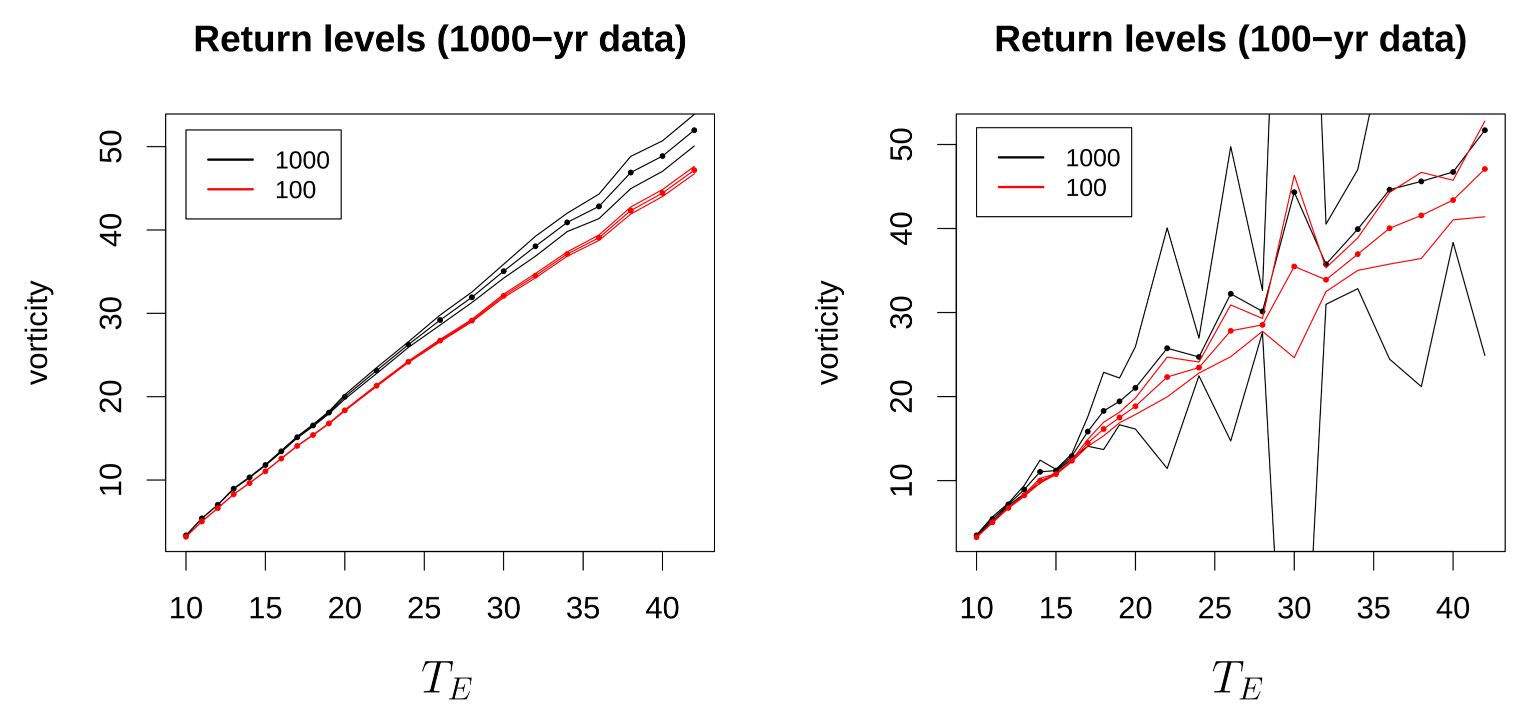
Robust extremes

Generate time series of length 10^3yrs length (recorded every 6hrs). Extract maxima over blocks of 1yr. Fit generalised extreme value (GEV) distribution:

$$G(x; \mu, \sigma, \xi) = \exp\left[-\left(1 + \xi\frac{x - \mu}{\sigma}\right)_+^{-1/\xi}\right]$$



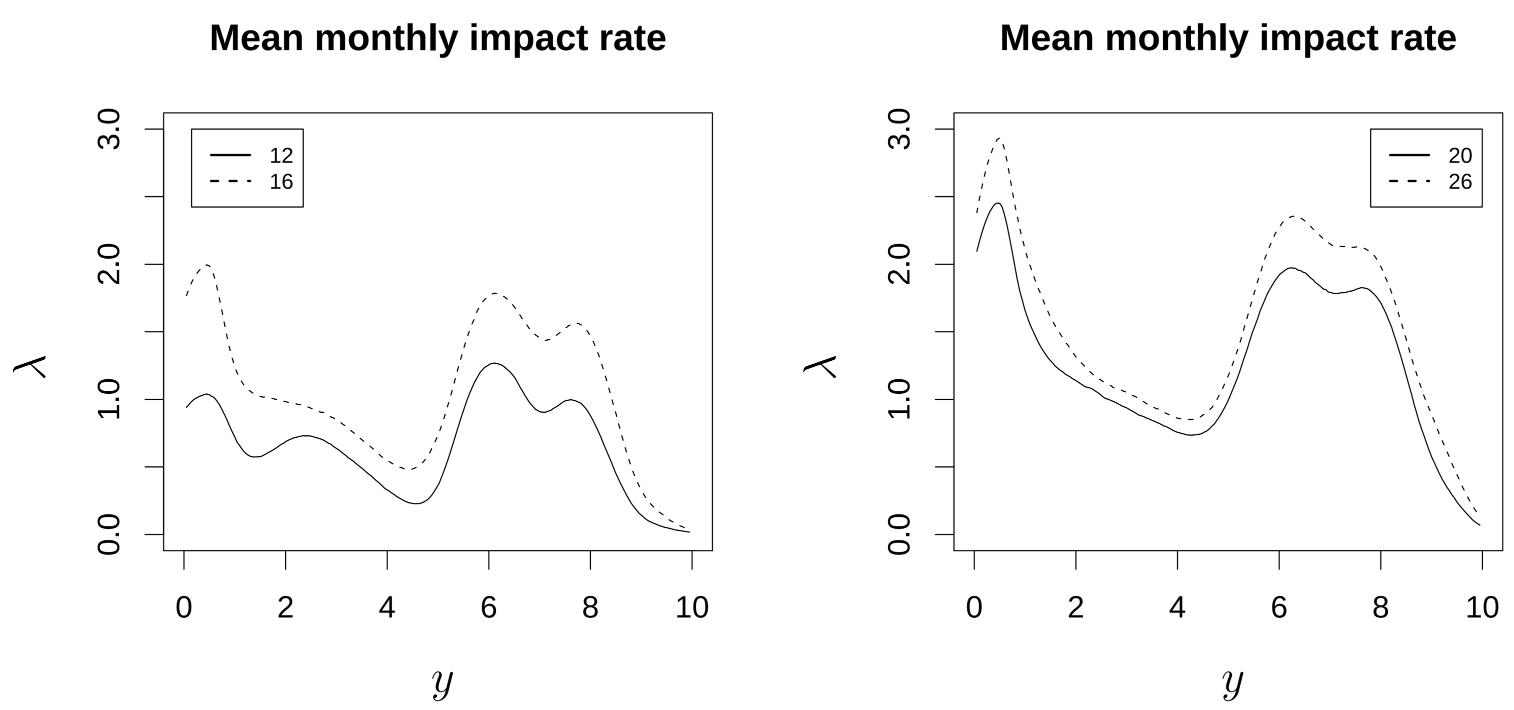
GEV parameters of maximum vorticity estimated from sequences of 100, 1000 maxima (red, black). Green: linear regression fit of the GEV parameters against T_E. Point estimates of ξ with 100 maxima oscillate wildly around "true" values and have very large confidence intervals (not shown).



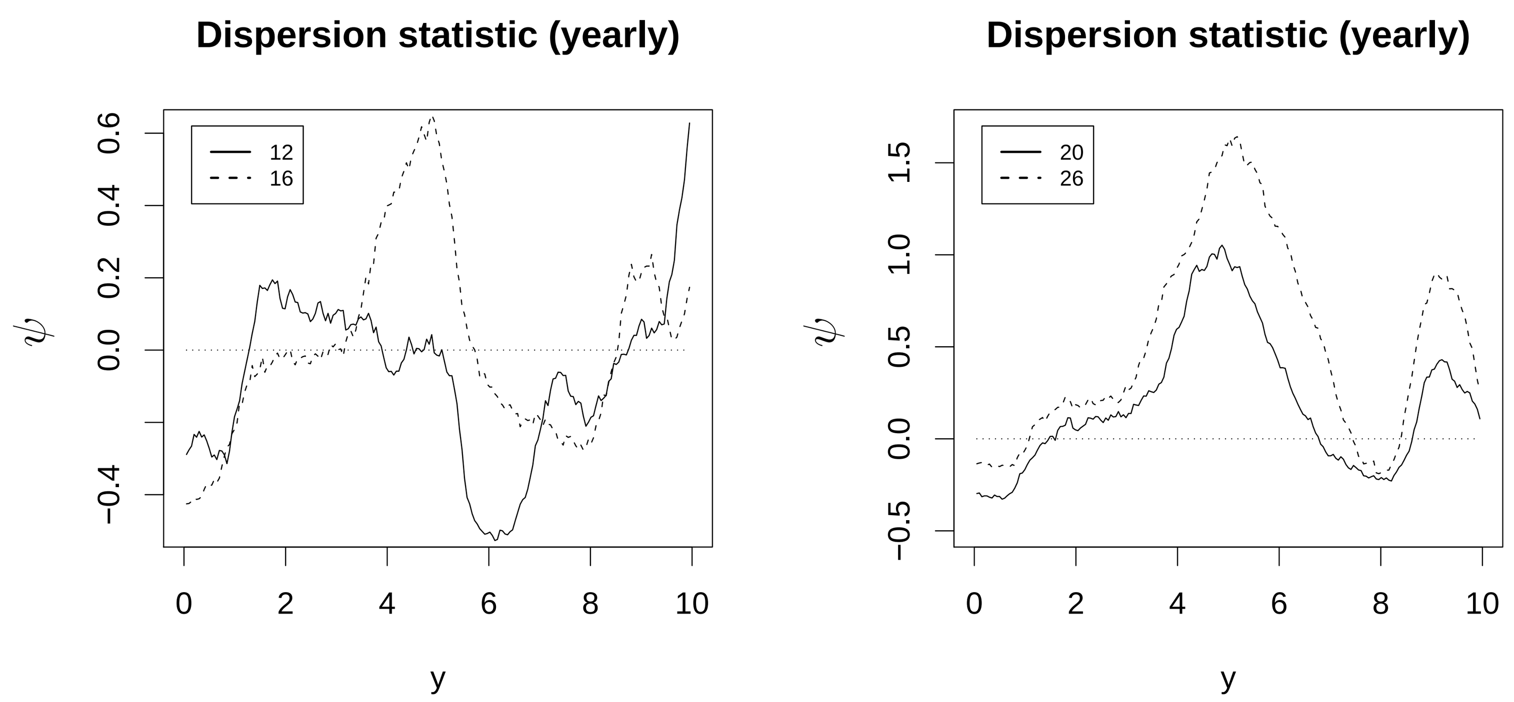
100yr and 1000yr return levels of vorticity, estimated with 1000 maxima (left) and 100 maxima (right). Huge confidence intervals for the 1000yr return levels extrapolated from 100 maxima.
Detecting robust dependence of extremes on T_E may require long datasets.
Robustness $\rightsquigarrow$ improved predictions by "pooling" data.

Vortex clustering

Automatic vortex tracking by simple algorithm (nearest-neighbour search)

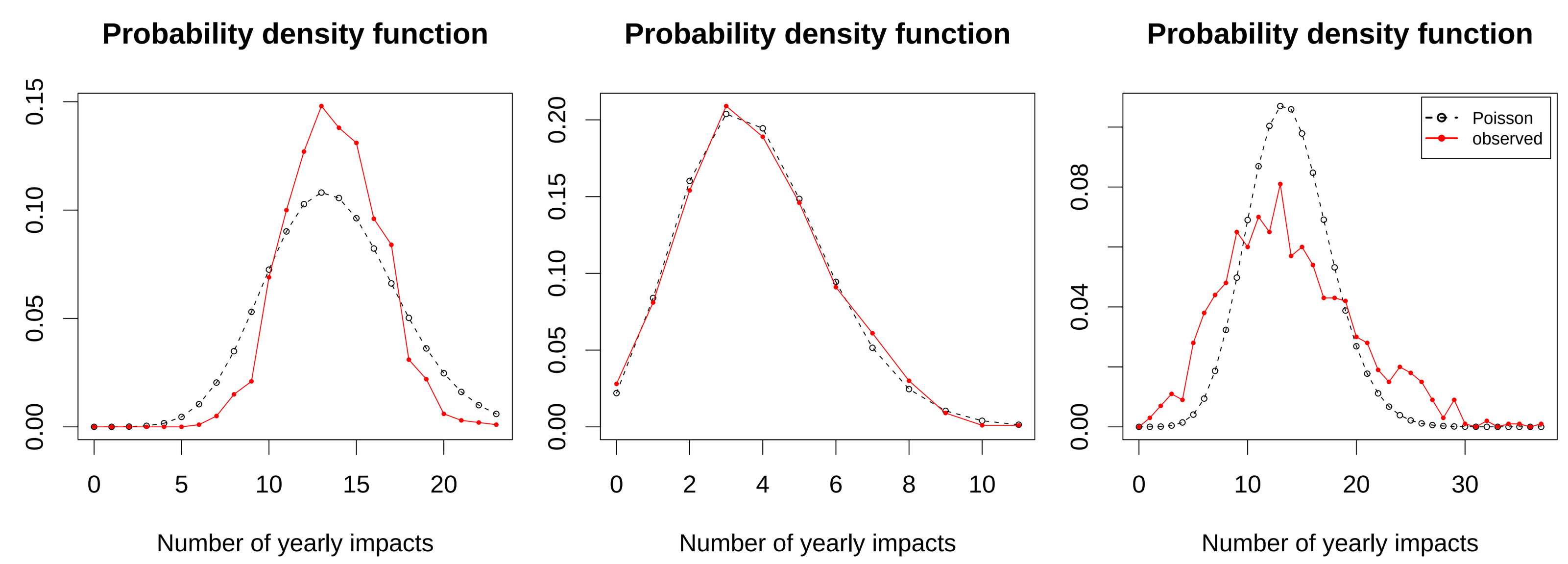


Monthly transit rate λ of vortex tracks on latitudinal "barrier" centered at longitude $x = L_x/2$, for $T_E = 12, 18$ (left) and $T_E = 20, 26$ (right).
Rate increases with T_E. Note the *vortex jet* for $6 \leq y \leq 8$.



Dispersion $\psi = \mathbb{V}ar(Y)/\mathbb{E}(Y) - 1$ of the yearly counts Y of vortex transits (same barrier and T_E values as above).
Underdispersion ($\psi < 0$, associated with regular behaviour in time) within vortex jet. Overdispersion ($\psi > 0$, associated with temporal *clustering*) for large T_E at southern boundary of *vortex jet*.

To be or not to be Poissonian



Empirical probability density of the yearly vortex transit counts for $(y, T_E) = (6, 12)$ (left), $(5, 12)$ (centre) and $(5, 26)$ (right), with Poisson fits (dashed).
Vortex transit process can be underdispersed (left), equidispersed (centre) and overdispersed (right), depending on latitude y and on baroclinic forcing T_E.