Appendix to: Envelope quasi-solitons in an excitable system with cross-diffusion

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(Dated: July 27, 2011)

Simulations

We use first order time stepping, fully explicit in the reaction terms and fully implicit in the cross-diffusion terms, with a second-order central difference approximation for the spatial derivatives and no-flux boundary conditions. We used steps $\Delta x = 1/10$ and $\Delta t = 1/5000$ for FHN kinetics (2) and $\Delta x = 1/\sqrt{20}$ and $\Delta t = 1/1000$ for LE kinetics (3). Initial conditions were set as $u(x, 0) = u_* + \mathrm{H}(\delta - x)$, $v(x, 0) = v_*$, to initiate a wave starting from the left end of the domain. Here (u_*, v_*) is the resting state, $(u_*, v_*) = (0, 0)$ for FHN and $(u_*, v_*) = (A/5, 1 + A^2/25)$ for LE, $\mathrm{H}()$ is the Heaviside function, and the wave seed length was typically chosen as $\delta = 2$. The interval length L was chosen sufficiently large, say for (2) typically at least $L = 350$, to allow wave propagation unaffected by boundaries, for some significant time.

Infinite line and centre of mass

To simulate propagation "on an infinite line", $L = \infty$, for fig. 1, we instantanously translated the solution by $\delta x_1 = 30$ away from the boundary each time the pulse, as measured at the level $u = 0.1$, approached the boundary to a distance smaller than $\delta x_2 = 100$, and filled in the new interval of x values by extending the u and v variables at constant levels. In these $L = \infty$ simulations, we defined the "center of mass" coordinate of the quasi-soliton solution as

$$x_c(t) = \left(\int_0^L (v(x,t) - v_*)^2 \,\mathrm{d}x \right)^{-1} \int_0^L x(v(x,t) - v_*)^2 \,\mathrm{d}x,$$

and used that for visualization, to align profiles recorded at different time moments.

Counting wavelets

For fig. 3(a), we counted peaks (wavelets) in the EQS solutions as the number of continuous intervals of x where $u > 0.1$. At some values of a, this number varied with time, as the shape of EQS changed while propagating, hence two different numbers of peaks for some values of a in fig. 3(a).

Fitting

We took the v-component of the given solution in the interval and selected the connected area in the (x, t) plane where $|v(x, t)| < 0.1$ ahead of the main wave. We numerically fitted this grid function $v(x, t)$ to (4) using Gnuplot implementation of Marquardt-Levenberg algorithm. The initial guess for parameters C, μ, c, k, x, ω was done "by eye". The fitting was initially on a small interval $t \in [5001, 5001.2]$ and then gradually extended to the interval $t \in [5001, 5015]$ in steps of 0.2, so that the result of one fitting was used as the initial guess for the next fitting. If we accept the fitted values for k and μ, then (5) gives a quadratic equation for λ, and its root with a positive real part gives $c \approx 4.07909$, $\omega = 6.15905$, which are in an agreement with the actual fitted values to 3 s.f.