Errata

V.N. Biktashev & A.V. Holden "Re-entrant activity and its control by resonant drift in a two-dimensional model of isotropic homogeneous ventricular tissue" *Proc Roy Soc London B* **263**: 1373–1382, 1996

This is the list of equations we actually used in our numerics. Differences from the printed version are marked by colour.

A Appendix. Equations of Excitability of a Single Guinea Pig Ventricular Cell

A.1 Units

s	second	time
μm	micrometer	space
μl	microlitre	volume
mJ	millijoule	energy
C	coulomb	electric charge
mV	millivolt	potential
nA	nanoampere	current
°K	kelvin	temperature
μS	microsiemen	conductance
μF	microfarad	capacitance
mol	mole	amount of substance
mM	mole per litre	concentration

A.2 Independent Dynamic Variables

- V - transmembrane voltage, mV
- m, h, d, x, f, q, r, $f_{\rm act}$, $f_{\rm prod}$ - gating variables, $0 \ldots 1$
- $[\mathrm{Na^+}]_\mathrm{i}$, $[\mathrm{K^+}]_\mathrm{i}$, $[\mathrm{Ca^{2+}}]_\mathrm{i}$ - intracellular ion concentrations, mM
- $[\mathrm{Ca^{2+}}]_\mathrm{up}$, $[\mathrm{Ca^{2+}}]_\mathrm{rel}$, $[\mathrm{Ca^{2+}}]_\mathrm{calmod}$, $[\mathrm{Ca^{2+}}]_\mathrm{trop}$ - intracellular partial $[\mathrm{Ca^{2+}}]$ concentrations, mM

A.3 Differential equations

$$
\begin{aligned}
\dot{V} &= \frac{-1}{C}(I_{\mathrm{K}} + I_{\mathrm{K1}} + I_{\mathrm{to}} + I_{\mathrm{siK}} + I_{\mathrm{bK}} + I_{\mathrm{NaK}} + I_{\mathrm{Na}} + I_{\mathrm{bNa}} + \\
&\quad I_{\mathrm{siNa}} + I_{\mathrm{NaCa}} + I_{\mathrm{siCa}} + I_{\mathrm{bCa}}) \\
\dot{m} &= \frac{200(V+41)}{1-\exp\left[-0.1(V+41)\right]}(1-m) - 8000\exp\left[-0.056(V+66)\right] m \\
\dot{h} &= 20\exp\left[-0.125(V+75)\right](1-h) - \frac{2000}{1+320\exp\left[-0.1(V+75)\right]}h \\
\dot{d} &= \frac{90(V+19)}{1-\exp\left[-(V+19)/4\right]}(1-d) - \frac{36(V+19)}{\exp\left[(V+19)/10\right]-1}d \\
\dot{x} &= \frac{0.5\exp\left[0.0826(V+50)\right]}{1+\exp\left[0.057(V+50)\right]}(1-x) - \frac{1.3\exp\left[-0.06(V+20)\right]}{1+\exp\left[-0.04(V+20)\right]}x \\
\dot{f} &= \frac{3.125(V+34)}{\exp\left[(V+34)/4\right]-1}(1-f) - \frac{25}{1+\exp\left[-(V+34)/4\right]}f \\
\dot{q} &= 333\left(\frac{1}{1+\exp\left[-(V+4)/5\right]} - q\right) \\
\dot{r} &= 0.033\exp\left[-V/17\right](1-r) - \frac{33}{1+\exp\left[-(V+10)/8\right]}r
\end{aligned}
$$

$$
\begin{aligned}
[\dot{\mathrm{Na}^{+}}]_{\mathrm{i}} &= \frac{-1}{V_{\mathrm{i}}F}\left(I_{\mathrm{Na}} + I_{\mathrm{bNa}}\frac{[\mathrm{Na}^{+}]_{\mathrm{o}}}{140} + 3I_{\mathrm{NaK}} + 3I_{\mathrm{NaCa}} + I_{\mathrm{siNa}}\right) \\
[\dot{\mathrm{K}^{+}}]_{\mathrm{i}} &= \frac{-1}{V_{\mathrm{i}}F}(I_{\mathrm{K}} + I_{\mathrm{K1}} + I_{\mathrm{siK}} + I_{\mathrm{bK}} + I_{\mathrm{to}} - 2I_{\mathrm{NaK}}) \\
[\dot{\mathrm{Ca}^{2+}}]_{\mathrm{i}} &= \frac{-1}{2V_{\mathrm{i}}F}(I_{\mathrm{siCa}} + I_{\mathrm{bCa}} - 2I_{\mathrm{NaCa}}) - I_{\mathrm{up}} + I_{\mathrm{rel}}\frac{V_{\mathrm{SRup}}V_{\mathrm{rel}}}{V_{\mathrm{i}}V_{\mathrm{up}}} \\
&\quad - \frac{\mathrm{d}}{\mathrm{dt}}[\mathrm{Ca}^{2+}]_{\mathrm{calmod}} - \frac{\mathrm{d}}{\mathrm{dt}}[\mathrm{Ca}^{2+}]_{\mathrm{trop}}
\end{aligned}
$$

$$
\begin{aligned}
[\dot{\mathrm{Ca}^{2+}}]_{\mathrm{up}} &= \frac{V_{\mathrm{i}}}{V_{\mathrm{SRup}}}I_{\mathrm{up}} - I_{\mathrm{tr}} \\
[\dot{\mathrm{Ca}^{2+}}]_{\mathrm{rel}} &= \frac{V_{\mathrm{up}}}{V_{\mathrm{rel}}}I_{\mathrm{tr}} - I_{\mathrm{rel}} \\
[\mathrm{Ca}^{2+}]^{\cdot}_{\mathrm{calmod}} &= 10^{5}(M_{\mathrm{trop}} - [\mathrm{Ca}^{2+}]_{\mathrm{calmod}})[\mathrm{Ca}^{2+}]_{\mathrm{i}} - 50[\mathrm{Ca}^{2+}]_{\mathrm{calmod}} \\
[\dot{\mathrm{Ca}^{2+}}]_{\mathrm{trop}} &= 10^{5}(C_{\mathrm{trop}} - [\mathrm{Ca}^{2+}]_{\mathrm{trop}})[\mathrm{Ca}^{2+}]_{\mathrm{i}} - 200[\mathrm{Ca}^{2+}]_{\mathrm{trop}}
\end{aligned}
$$

$$\dot{f_{act}} = (1 - f_{act} - f_{prod})(500(\frac{[Ca^{2+}]_i}{[Ca^{2+}]_i + k_{mCa}\,\theta(I_{siCa})})^2) - f_{act}(500(\frac{[Ca^{2+}]_i}{[Ca^{2+}]_i + k_{mCa}\,\theta(I_{siCa})})^2 + 60)$$

$$\dot{f_{prod}} = f_{act}(500(\frac{[Ca^{2+}]_i}{[Ca^{2+}]_i + k_{mCa}\,\theta(I_{siCa})})^2 + 60) - f_{prod}$$

where

$$\theta(I_{siCa}) = \begin{cases} 1 & \text{if } I_{siCa} > -0.5 \\ 0.1 & \text{otherwise} \end{cases}$$

A.4 Dependent Quantities - Functions of Dynamic Variables

A.4.1 Channel Transmembrane Currents

$$I_K = \frac{xI_{Kmax}}{140}([K^+]_i - [K^+]_o \exp\left[\frac{-V}{RT/F}\right])$$

$$I_{K1} = G_{K1}\frac{[K^+]_o}{[K^+]_o + k_{mK1}}(\frac{V - E_K}{1 + \exp\left[\frac{V - E_K + 10 - V_{shift}}{RT/2F}\right]})$$

$$I_{to} = G_{to}(V - E_K)qr$$

$$I_{bK} = G_{bK}(V - E_K)$$

$$I_{Na} = G_{Na}(V - E_{mh})m^3h$$

$$I_{bNa} = G_{bNa}(V - E_{Na})$$

$$I_{siCa} = 4P_{Ca}df\frac{\frac{V-50}{RT/F}}{1 - \exp\left[\frac{-(V-50)}{RT/2F}\right]}[[Ca^{2+}]_i \exp\left[\frac{50}{RT/2F}\right] - [Ca^{2+}]_o \exp\left[\frac{-(V-50)}{RT/2F}\right]]$$

$$I_{siK} = P_{CaK}P_{Ca}df\frac{\frac{V-50}{RT/F}}{1 - \exp\left[\frac{-(V-50)}{RT/F}\right]} [[K^+]_i \exp\left[\frac{50}{RT/F}\right] - [K^+]_o \exp\left[\frac{-(V-50)}{RT/F}\right]]$$

$$I_{siNa} = P_{CaNa}P_{Ca}df\frac{\frac{V-50}{RT/F}}{1 - \exp\left[\frac{-(V-50)}{RT/F}\right]} [[Na^+]_i \exp\left[\frac{50}{RT/F}\right] - [Na^+]_o \exp\left[\frac{-(V-50)}{RT/F}\right]]$$

$$I_{\mathrm{bCa}} = G_{\mathrm{bCa}}(V - E_{\mathrm{Ca}})$$

A.4.2 Pump/Exchanger Transmembrane Currents

$$I_{\mathrm{NaK}} = I_{\mathrm{NaKmax}} \frac{[\mathrm{K}^+]_\mathrm{o}}{[\mathrm{K}^+]_\mathrm{o} + k_{\mathrm{mK}}} \frac{[\mathrm{Na}^+]_\mathrm{i}}{[\mathrm{Na}^+]_\mathrm{i} + k_{\mathrm{mNa}}}$$

$$I_{\mathrm{NaCa}} = k_{\mathrm{NaCa}} \frac{\exp\left[\gamma \frac{V}{RT/F}\right] [\mathrm{Na}^+]_\mathrm{i}{}^3 [\mathrm{Ca}^{2+}]_\mathrm{o} - \exp\left[-(1-\gamma)\frac{V}{RT/F}\right] [\mathrm{Na}^+]_\mathrm{o}^3 [\mathrm{Ca}^{2+}]_\mathrm{i}}{1 + d_{\mathrm{NaCa}}([\mathrm{Ca}^{2+}]_\mathrm{i} [\mathrm{Na}^+]_\mathrm{o}{}^3 + [\mathrm{Ca}^{2+}]_\mathrm{o} [\mathrm{Na}^+]_\mathrm{i}{}^3)},$$

A.4.3 $[\mathrm{Ca}^{2+}]$ Sequestration Flows

$$I_{\mathrm{up}} = \frac{0.4\,[\mathrm{Ca}^{2+}]_\mathrm{i} - 0.03\,[\mathrm{Ca}^{2+}]_{\mathrm{up}} \frac{k_{\mathrm{cyca}} k_{\mathrm{xcs}}}{k_{\mathrm{srca}}}}{[\mathrm{Ca}^{2+}]_\mathrm{i} + [\mathrm{Ca}^{2+}]_{\mathrm{up}} \frac{k_{\mathrm{cyca}} k_{\mathrm{xcs}}}{k_{\mathrm{srca}}} + k_{\mathrm{cyca}} k_{\mathrm{xcs}} + k_{\mathrm{cyca}}}$$

$$I_{\mathrm{tr}} = 50([\mathrm{Ca}^{2+}]_{\mathrm{up}} - [\mathrm{Ca}^{2+}]_{\mathrm{rel}})$$

$$I_{\mathrm{rel}} = (\frac{f_{\mathrm{act}}}{f_{\mathrm{act}} + 0.25})^2 k_{\mathrm{mCa2}} [\mathrm{Ca}^{2+}]_{\mathrm{rel}}$$

A.4.4 Reversal potentials

$$E_{\mathrm{Na}} = \frac{RT}{F} \log(\frac{[\mathrm{Na}^+]_\mathrm{o}}{[\mathrm{Na}^+]_\mathrm{i}})$$

$$E_{\mathrm{K}} = \frac{RT}{F} \log(\frac{[\mathrm{K}^+]_\mathrm{o}}{[\mathrm{K}^+]_\mathrm{i}})$$

$$E_{\mathrm{Ca}} = \frac{RT}{2F} \log(\frac{[\mathrm{Ca}^{2+}]_\mathrm{o}}{[\mathrm{Ca}^{2+}]_\mathrm{i}})$$

$$E_{\mathrm{mh}} = \frac{RT}{F} \log(\frac{[\mathrm{Na}^+]_\mathrm{o} + 0.12[\mathrm{K}^+]_\mathrm{o}}{[\mathrm{Na}^+]_\mathrm{i} + 0.12[\mathrm{K}^+]_\mathrm{i}})$$

A.5 Standard Parameter Values

C	$200 \cdot 10^{-6}\mu$F	k_{NaCa}	$5 \cdot 10^{-4} nA$
I_{Kmax}	1.0nA	d_{NaCa}	0.0
k_{mK1}	10mM	γ	$\frac{1}{2}$
k_{mK}	1mM	k_{cyca}	$3 \cdot 10^{-4}$mM
k_{mNa}	40mM	k_{xcs}	0.4mM
k_{mCa}	$5 \cdot 10^{-4}$mM	k_{srca}	0.5mM
V_{shift}	20.0mV	F	96485C/mol
I_{NaKmax}	0.7nA	R	8314.41mJ/(mol°K)
G_{Na}	2.5μS	T	310°K
G_{to}	0.005μS	V_{ecs}	0.4
G_{bK}	0.0006μS	$radius$	15μm
G_{K1}	1.0μS	$length$	80μm
G_{bNa}	0.0006 μS	V_{cell}	$\pi\ radius^2 length \cdot 10^{-9}, \mu$l
G_{bCa}	0.00025μS	V_{i}	$(1 - V_{\mathrm{ecs}} - V_{\mathrm{up}} - V_{\mathrm{rel}})V_{\mathrm{cell}}, \mu$l
P_{Ca}	0.25nA/mM	V_{up}	0.01
P_{CaK}	0.002	V_{rel}	0.1
P_{CaNa}	0.002	V_{SRup}	$V_{\mathrm{cell}} V_{\mathrm{up}} \mu$l
$[\mathrm{Ca}^{2+}]_{\mathrm{o}}$	2mM	k_{mCa2}	250 nA/mM
$[\mathrm{K}^{+}]_{\mathrm{o}}$	4mM	M_{trop}	0.02mM
$[\mathrm{Na}^{+}]_{\mathrm{o}}$	140mM	C_{trop}	0.05mM